

Exercise 18

Find dy/dx by implicit differentiation.

$$x \sin y + y \sin x = 1$$

Solution

Differentiate both sides with respect to x .

$$\frac{d}{dx}(x \sin y + y \sin x) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(x \sin y) + \frac{d}{dx}(y \sin x) = 0$$

$$\left[\frac{d}{dx}(x) \right] \sin y + x \left[\frac{d}{dx}(\sin y) \right] + \left[\frac{d}{dx}(y) \right] \sin x + y \left[\frac{d}{dx}(\sin x) \right] = 0$$

$$(1) \sin y + x \left[(\cos y) \cdot \frac{d}{dx}(y) \right] + (y') \sin x + y(\cos x) = 0$$

$$\sin y + xy' \cos y + y' \sin x + y \cos x = 0$$

Solve for y' .

$$y'(x \cos y + \sin x) = -\sin y - y \cos x$$

$$y' = -\frac{\sin y + y \cos x}{x \cos y + \sin x}$$